Algebra

Seeing Structure in Expressions

Interpret the structure of expressions.

1 Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.1

Complexity a

a Represent a realworld situation with an expression, both numerals and variables. Recognize parts of the expression in the real-world situation. A.SSE.1.A

Complexity b

b Represent a real-world situation with a numeric expression. Recognize parts of the expression in the real-world situation. A.SSE.1.B

Complexity c

c Represent a realworld situation with a model using concrete objects. A.SSE.1.C

Learning Progression

- Demonstrate or act out the situation A.SSE.1.LP.A
- Show what addition, subtraction, multiplication and division represents using manipulatives. A.SSE.1.LP.B
- Recognize the symbols for addition (+), subtraction, (–), multiplication (×), division (÷), and equals (=). A.SSE.1.LP.C
- Engagement Statements (demonstration of engaged in the topic) A.SSE.1.LP.D
- Interact with concrete objects. A.SSE.1.LP.E
- Interact with models. A.SSE.1.LP.F

2 Use the structure of an expression to identify ways to rewrite it. For example, to factor 3x(x − 5) + 2(x − 5), students should recognize that the "x − 5" is common to both expressions being added, so it simplifies to (3x + 2)(x − 5); or see x4 − y4 as (x2)2 − (y2)2 , thus recognizing it as a difference of squares that can be factored as (x2 − y2)(x2 + y2). A.SSE.2

Complexity a

a Simplify expressions involving variables (e.g., (2(x + 4) = 2x + 8)). A.SSE.2.A

Complexity b

b Identify the equivalent numeric expression (e.g., 7 + 5 = 5 + 7). A.SSE.2.B

Complexity c

c Identify equivalent expressions with whole numbers less than 10 using concrete objects (e.g., objects, dots, etc.). A.SSE.2.C

Learning Progression

Not on BP

Write expressions in equivalent forms to solve problems.

3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. (A1, M2) b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (A1, M2) c. Use the properties of exponents to transform expressions for exponential functions. For example, 8t can be written as 23t. A.SSE.3

Complexity a

a Apply properties of integer exponents to generate equivalent variable expressions (e.g., b2 x b4 = b6). A.SSE.3.A

Complexity b

b Apply properties of integer exponents to generate equivalent numerical expressions (e.g., 52 x 54 = 56). A.SSE.3.B

Complexity c

c Interpret numerical expressions with exponents (e.g., 54 means 5 x 5 x 5 x 5). A.SSE.3.C

Learning Progression

- Demonstrate or act out the situation. A.SSE.3.LP.A
- Show what addition, subtraction, multiplication and division represents using manipulatives. A.SSE.3.LP.B
- Identify the base and the exponent of an exponential expression. A.SSE.3.LP.C
- Recognize the symbols for addition (+), subtraction, (–), multiplication (×), division (÷), and equals (=). A.SSE.3.LP.D
- Count physical objects up to 30. A.SSE.3.LP.E
- Engagement Statements (demonstration of engaged in the topic) A.SSE.3.LP.F
- Interact with concrete objects A.SSE.3.LP.G
- Interact with models and pictures. A.SSE.3.LP.H
- Interact with representations of the unknown. A.SSE.3.LP.I

Arithmetic with Polynomials and Rational Expression Standards

Perform arithmetic operations on polynomials.

1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. a. Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2) b. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3) A.APR.1

Complexity a

a Add and subtract linear and/or quadratic polynomials. Models may be used. A.APR.1.A

Complexity b

b Add and subtract linear polynomials. Models may be used. A.APR.1.B

Complexity c

c Add linear polynomials. Models may be used. A.APR.1.C

Learning Progression

Not on BP

Understand the relationship between zeros and factors of polynomials.

2 Understand and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x − a is p(a). In particular, p(a) = 0 if and only if (x − a) is a factor of p(x). A.APR.2

Complexity a

a Multiply two binomials. A.APR.2.A

Complexity b

b Multiply a variable by a binomial. A.APR.2.B

Complexity c

c Identify a polynomial (binomials only). A.APR.2.C

Learning Progression

Not on BP

3 Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial. A.APR.3

Complexity a

a Find the zeros of a polynomial when the polynomial is factored (e.g., x2 – 9 = 0 and x2 = 3x = 2 = 0). A.APR.3.A

Complexity b

b Identify a polynomial (trinomial) (e.g., x2 = 3x = 2 = 0). A.APR.3.B

Complexity c

c Identify a polynomial (binomial) (e.g., x2 – 9 = 0). A.APR.3.C

Learning Progression

Not on BP

Rewrite rational expressions.

6 Rewrite simple rational expressions in different forms; write a(x)/ b(x) in the form q(x) + r(x)/ b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. A.APR.6

Complexity a

a Identify a rational expression (e.g., 6x/ 3 = 2x). A.APR.6.A

Complexity b

b Rewrite expressions in different forms (e.g., x2 + 1 = (x * x) + 1).v A.APR.6.B

Complexity c

c Given a visual model, identify an expression (e.g. 2 * 2 * 2 = 23). A.APR.6.C

Learning Progression

Not on BP

Creating Equations Standards

Create equations that describe numbers or relationships.

1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. a. Focus on applying linear and simple exponential expressions. (A1, M1) b. Focus on applying simple quadratic expressions. (A1, M2) c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3) A.CED.1

Complexity a

a Represent and solve a realworld situation with a two-step linear equation or inequality (e.g., Abby has $15 to spend on a snack and two matching T-shirts. If she spends $3 on the snack, what is the maximum price of each T-shirt? Key: $x \leq 5$). A.CED.1.A

Complexity b

b Represent and solve a real-world problem with a onestep linear equation or inequality (e.g., Abby has $5, and she wants to buy a T-shirt for $8. How much more money does she need? Key: $5 + x = 8$) A.CED.1.B

Complexity c

c Represent a realworld problem with a linear equation, using concrete objects, models and pictures (see example below). A.CED.1.C

Learning Progression

- Demonstrate or act out the situation. A.CED.1.LP.A
- Recognize the unknown. A.CED.1.LP.B
- Show what addition, subtraction, multiplication and division represents using manipulatives. A.CED.1.LP.C
- Know that a symbol ☒ or letter can represent a missing value. A.CED.1.LP.D
- Count physical objects up to 30. A.CED.1.LP.E
- Relate a picture or objects to a number sentence. A.CED.1.LP.F
- Engagement Statements (demonstration of engaged in the topic) A.CED.1.LP.G
- Interact with concrete objects. A.CED.1.LP.H
- Represent numbers. A.CED.1.LP.I
- Interact with representations of the unknown. A.CED.1.LP.J

2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. a. Focus on applying linear and simple exponential expressions. (A1, M1) b. Focus on applying simple quadratic expressions. (A1, M2) c. Extend to include more complicated function situations with the option to graph with technology. (A2, M3) A.CED.2

Complexity a

a Create an equation with two variables to represent a linear relationship between quantities in a given context (e.g., $y = 2x + 4$). A.CED.2.A

Complexity b

b Using a two-variable equation describing a realworld situation, given the value of one variable, find and interpret the value of the other variable (e.g., Sally starts with $4 and gets an allowance of $2 each week. After x weeks, she has y = 2x + 4 dollars. When x = 3, find y and interpret the result). A.CED.2.B

Complexity c

c Identify the meaning of each number and/ or variable in a given two-variable equation that describe a realworld situation (e.g., Sally starts with $4 and gets an allowance of $2 each week. After x weeks she has y = 2x + 4 dollars. What does 4 represent? What does x represent?). A.CED.2.C

Learning Progression

- Recognize variables. A.CED.2.LP.A
- Recognize that one variable affects the other variable. A.CED.2.LP.B
- Recognize that a variable can represent any number. A.CED.2.LP.C
- Demonstrate that a coefficient represents a constant decrease or increase. A.CED.2.LP.D
- Demonstrate or act out the situation. A.CED.2.LP.E
- Recognize the unknown. A.CED.2.LP.F
- Show what addition, subtraction, multiplication and division represents using manipulatives. A.CED.2.LP.G
- Know that a symbol □ or letter can represent a missing value. A.CED.2.LP.H
- Engagement Statements (demonstration of engaged in the topic) A.CED.2.LP.I
- Interact with concrete objects. A.CED.2.LP.J
- Represent quantities. A.CED.2.LP.K
- Interact with representations of the unknown. A.CED.2.LP.L

3 Represent constraints by equations or inequalities, and by systems of equations and/ or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (A1, M1) a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3) A.CED.3

Complexity a

a Represent a constraint with an equation or inequality in two variables (e.g., x + y ≤ 8, describing the number of boys and girls in an 8-passenger van). A.CED.3.A

Complexity b

b Create a one-variable constraint using an inequality (e.g., x ≤ 6). A.CED.3.B

Complexity c

c Demonstrate a constraint using words or models (e.g., how many students can fit at this table?). A.CED.3.C

Learning Progression

- [Between c and b: A.CED.3.LP.A
- Identify the symbols <, >, and =.] A.CED.3.LP.B
- Recognize the unknown.v A.CED.3.LP.C
- Demonstrate quantities using manipulatives. A.CED.3.LP.D
- Understand the order of numbers. A.CED.3.LP.E
- Identify limitations of a real-world situation. A.CED.3.LP.F
- Compare quantities using manipulatives or pictures to identify which is more and which is less. A.CED.3.LP.G
- Engagement Statements (demonstration of engaged in the topic) A.CED.3.LP.H
- Demonstrate or act out a situation. A.CED.3.LP.I
- Interact with real-world situations with restraints, e.g., seats at a table, passengers in a bus, eggs in a carton. A.CED.3.LP.J
- Interact with situations involving greater than, equal to, or less than. A.CED.3.LP.K
- Interact with a model of a real-world situation. A.CED.3.LP.L

4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. a. Focus on formulas in which the variable of interest is linear or square. For example, rearrange Ohm's law V = IR to highlight resistance R, or rearrange the formula for the area of a circle A = (π)r2 to highlight radius r. (A1) b. Focus on formulas in which the variable of interest is linear. For example, rearrange Ohm's law V = IR to highlight resistance R. (M1) c. Focus on formulas in which the variable of interest is linear or square. For example, rearrange the formula for the area of a circle A = (π)r2 to highlight radius r. (M2) d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3) A.CED.4

Complexity a

a Rearrange a onestep formula to highlight a quantity (e.g., use the formula a=lw to highlight the length of the rectangle by rearranging it to l=a/w). A.CED.4.A

Complexity b

b Rearrange a one-step equation to solve for a variable (e.g., solve for x: y=2+x, y/2=x). A.CED.4.B

Complexity c

c Match a formula to a given situation (e.g., recognize that a=lw is the formula for the area of a rectangle) A.CED.4.C

Learning Progression

- Recognize that a formula represents a situation. A.CED.4.LP.A
- Recognize what the variables represent in the formula. A.CED.4.LP.B
- Model a situation with manipulatives. A.CED.4.LP.C
- Relate a picture or objects to a number sentence. A.CED.4.LP.D
- Read and interpret a traditional one-step number sentence (2 × 3 = □). A.CED.4.LP.E
- Identify a number sentence. A.CED.4.LP.F
- Recognize the symbols for addition (+), subtraction, (–), multiplication (×), division A.CED.4.LP.G
- (÷), and equals (=). A.CED.4.LP.H
- Recognize a numerical expression with and without variables. A.CED.4.LP.I
- Demonstrate or act out a situation. A.CED.4.LP.J
- Engagement Statements (demonstration of engaged in the topic) A.CED.4.LP.K
- Interact with real-world situations. A.CED.4.LP.L
- Interact with a model of a real-world situation. A.CED.4.LP.M
- Interact with representations of the unknown. A.CED.4.LP.N

Reasoning with Equations and Inequalities Standards

Understand solving equations as a process of reasoning and explain the reasoning.

1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.1

Complexity a

a Order a given sequence of steps to solve an equation (e.g., 2x + 5 = 13). Solve two-step equations with integer coefficients and solutions, explaining the steps. A.REI.1.A

Complexity b

b Determine a step needed to solve a two-step equation. A.REI.1.B

Complexity c

c Determine the step needed to solve a one-step equation (e.g., to solve x + 5 = 13, subtract 5 from both sides). A.REI.1.C

Learning Progression

- [Between C and B: A.REI.1.LP.A
- Identify the first step in solving a two-step problem involving addition and subtraction.] A.REI.1.LP.B
- Demonstrate the meaning of the solution. A.REI.1.LP.C
- Understand that the variable represents the unknown. A.REI.1.LP.D
- Understand that the operations addition and subtraction are inverse operations. A.REI.1.LP.E
- Understand the the operations multiplication and divisions are inverse operations. A.REI.1.LP.F
- Understand that the equal sign is a balance. A.REI.1.LP.G
- Understand that an operation applied to one side needs to be applied to the other. A.REI.1.LP.H
- Use manipulatives to understand the situation. A.REI.1.LP.I
- Demonstrate or act out a situation. A.REI.1.LP.J
- Engagement Statements (demonstration of engaged in the topic) A.REI.1.LP.K
- Interact with manipulatives that represent a situation. A.REI.1.LP.L
- Interact with real-world situations. A.REI.1.LP.M
- Interact with a model of a real-world situation. A.REI.1.LP.N
- Interact with representations of the unknown. A.REI.1.LP.O
- Interact with a balance scale. A.REI.1.LP.P

2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. A.REI.2

Complexity a

a Solve linear equations with more than one step. A.REI.2.A

Complexity b

b Solve 1-step linear equations. A.REI.2.B

Complexity c

c Solve for the missing number within a given number sentence involving addition or subtraction of numbers less than 10. A.REI.2.C

Learning Progression

Not on BP

Solve equations and inequalities in one variable.

3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. A.REI.3

Complexity a

a Solve a two- or three-step linear equation in one variable. Models may be used. A.REI.3.A

Complexity b

b Solve a one-step linear equation in one variable. Models may be used. A.REI.3.B

Complexity c

c Given a linear equation in one-variable and a list of possible solutions, identify the solution of the equation. A.REI.3.C

Learning Progression

- Understand the idea of what a solution to an equation means. A.REI.3.LP.A
- Identify the solution of the equation using manipulatives. A.REI.3.LP.B
- Using trial error explore the idea of the equal sign being a balance. A.REI.3.LP.C
- Identify a number sentence. A.REI.3.LP.D
- Recognize the symbols for addition (+), subtraction, (–), multiplication (×), division (÷), and equals (=). A.REI.3.LP.E
- Read and interpret a traditional one-step number sentence (2 × 3 = □). A.REI.3.LP.F
- Relate a picture or objects to a number sentence. A.REI.3.LP.G
- Know that a symbol □ or letter can represent a missing value. A.REI.3.LP.H
- Count to 30. A.REI.3.LP.I
- Count physical objects up to 30. A.REI.3.LP.J
- Recognize a numerical expression with and without variables. A.REI.3.LP.K
- Identify a numerical expression without exponents. A.REI.3.LP.L
- Engagement Statements (demonstration of engaged in the topic) A.REI.3.LP.M
- Interact with a balance scale. A.REI.3.LP.N
- Interact with physical objects (blocks) or drawings that represent an expression or an equation. A.REI.3.LP.O
- Interact with physical objects (blocks) or drawings representing addition, subtraction, or multiplication word problems. A.REI.3.LP.P

4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. b. Solve quadratic equations as appropriate to the

initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. A.REI.4

Complexity a

a Identify or create perfect squares (e.g., square root of 25 = 5). A.REI.4.A

Complexity b

b Identify equivalent expressions that are cubes (e.g., m3 = m x m x m). A.REI.4.B

Complexity c

c Identify equivalent expressions that are squared (e.g., m2 = m x m). A.REI.4.C

Learning Progression

Not on BP

Solve systems of equations.

6 Solve systems of linear equations algebraically and graphically. a. Limit to pairs of linear equations in two variables. (A1, M1) b. Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3) A.REI.6

Complexity a

a Identify the coordinate at which two lines intersect. A.REI.6.A

Complexity b

b Locate the point on the graph at which two lines intersect. A.REI.6.B

Complexity c

c Identify whether two lines intersect. A.REI.6.C

Learning Progression

- Identify a line. A.REI.6.LP.A
- Identify crossing lines in the real-world. A.REI.6.LP.B
- Show that the length of intersecting lines can be extended so they will eventually cross. A.REI.6.LP.C
- Engagement Statements (demonstration of engaged in the topic) A.REI.6.LP.D
- Interact with manipulatives representing lines. A.REI.6.LP.E

7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = −3x and the circle x2 + y2 = 3. A.REI.7

Complexity a

a Locate the coordinate of the point(s) at which a line intersects a quadratic function (e.g., at which two coordinates does the line intersect the parabola?). A.REI.7.A

Complexity b

b Locate the point(s) on the graph at which a line intersects a quadratic function (e.g., identify on the graph where the line intersects the parabola). A.REI.7.B

Complexity c

c Identify whether a line intersects a quadratic function (e.g., does the line intersect the parabola at one or two points? Does the line intersect the parabola?). A.REI.7.C

Learning Progression

Not on BP

Represent and solve equations and inequalities graphically

10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.10

Complexity a

a Given a graph and an equation, fill out three points on a corresponding table of values. A.REI.10.A

Complexity b

b Given a table of values, graph the line on the coordinate plane. A.REI.10.B

Complexity c

c Identify a point on a line on a coordinate plane. A.REI.10.C

Learning Progression

- Locate an ordered pair (x, y) as a point on the coordinate plane. A.REI.10.LP.A
- Plot a point on the coordinate plane. A.REI.10.LP.B
- Identify the x- and y- axis. A.REI.10.LP.C
- Identify that the x- and y- axes are number lines. A.REI.10.LP.D
- Identify a point. A.REI.10.LP.E
- Demonstrate that on a number line and on the coordinate plane the spaces need to be counted not the grid lines. A.REI.10.LP.F
- Identify a line. A.REI.10.LP.G
- Locate the origin at the point (0, 0) on the coordinate plane. A.REI.10.LP.H
- Engagement Statements (demonstration of engaged in the topic) A.REI.10.LP.I
- Interact with the coordinate grid. A.REI.10.LP.J

11 Explain why the x-coordinates of the points where the graphs of the equation y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, making tables of values, or finding successive approximations). A.REI.11

Complexity a

a Locate the coordinate at which two lines intersect. Using the x coordinate of the intersection point, substitute it back into the original equation to show that it is a solution of the equation. A.REI.11.A

Complexity b

b Locate the coordinate point on the graph at which two lines intersect. A.REI.11.B

Complexity c

c Identify whether two lines intersect. A.REI.11.C

Learning Progression

- Identify a line. A.REI.11.LP.A
- Identify crossing lines in the real-world. A.REI.11.LP.B
- Show that non-parallel lines can be extended so they eventually cross. A.REI.11.LP.C
- Engagement Statements (demonstration of engaged in the topic) A.REI.11.LP.D
- Interact with the coordinate grid. A.REI.11.LP.E
- Interact with manipulatives representing lines. A.REI.11.LP.F

12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. A.REI.12

Complexity a

a Given a graph of an inequality including the shaded region, identify three points that make the inequality true. A.REI.12.A

Complexity b

b Identify on a graph of a line ≤, ≥ is represented by a solid line; and < and > are represented by a dotted line. A.REI.12.B

Complexity c

c Identify the graph of a linear inequality has a shaded region. A.REI.12.C

Learning Progression

- Identify ordered pairs that satisfy an inequality A.REI.12.LP.A
- Plot ordered pairs that satisfy an inequality. A.REI.12.LP.B
- Understand that a linear inequality has more than one solution. A.REI.12.LP.C
- Understand that the solutions to a linear equation in two variables are all the points on a straight line. A.REI.12.LP.D
- Understand that the solutions to a linear inequality in two variables are points on a plane. A.REI.12.LP.E
- Each point on a coordinate plane is associated to a (x,y) ordered pair. A.REI.12.LP.F
- Using trial error explore ordered pairs that satisfy an inequality A.REI.12.LP.G
- Using trial error explore ordered pairs that satisfy an equation. A.REI.12.LP.H
- Identify a shaded region on a coordinate plane. A.REI.12.LP.I
- Identify a shaded region in a picture or of an object. A.REI.12.LP.J

- Engagement Statements (demonstration of engaged in the topic) A.REI.12.LP.K
- Interact with a graph A.REI.12.LP.L
- Interact with the coordinate grid. A.REI.12.LP.M